Estimating Pi: A Use Case of Quantum Accelerated Monte Carlo

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Abstract

Quantum Computing is a new paradigm of computation that brings in new capabilities that are not realizable with classical computers. However, harnessing these advantages is no easy task. One groundbreaking innovation is Quantum Accelerated Monte Carlo (QAMC), a method that reduces quadratically the number of oracle calls, or shots, required in Monte Carlo processes. The reduction in the amount of shots is of interest for the optimization of these processes, which tend to be computationally heavy. In this paper, we present an approach to a rather well-known problem, the estimation of π, through both classical and quantum Monte Carlo, only to improve the latter with QAMC. The results show how the same precision can be obtained with only a fraction of the shots in the original case.

1 Introduction

Quantum Computing (QC) is a computational paradigm based on the postulates of Quantum Mechanics. This shift from classical computing allows for new ways of both representing and managing information, but harnessing its computational advantages can be difficult. One of the most relevant methods for taking advantage of QC is Quantum Accelerated Monte Carlo (QAMC) [1], which reduces with a quadratic factor the number of shots that are required for obtaining an estimation through a Monte Carlo process. In this paper, we use this technique to estimate π, as a showcase of the potential of QC to efficiently solve problems of this nature.

2 Materials and methods

The classical approach of estimating π with Monte Carlo is based on the generation of random points $(x, y) \in \mathbb{R}^2$ in a square of side $2r$ with a inscribed circle of radius r. The relationship between their areas is $A_C/A_S = \pi r^2/4r^2 = \pi/4$, therefore $\pi = 4 \times A_C/A_S$; this proportion holds when we consider just a quarter of the construction. The generated points are taken as an approximation to these areas, as every point accounts for the area of the square, and any point (x, y) such that $x^2 + y^2 \leq r$ account for the area of the circle. With these considerations, π can be estimated as $\tilde{\pi} \approx 4 \times points\_in\_cirlce/points\_in\_square$. The more shots of the Monte Carlo experiment are run, the higher the precision of the estimation of π.

For the QC proposal, the construct is discretized in a grid of $n \times n$ parcels. Rather than generating random points, a pair of coordinates $(x_p, y_p) \in \{0, 1, \ldots, n-1\}^2$ is generated to select a random parcel. The parcel accounts for the circle if its center (x_{p_c}, y_{p_c}) holds that $x_{p_c}^2 + y_{p_c}^2 \leq r$. Figure 1 illustrates several examples. It should be noted that this discretization is no different than the discretization caused by the limited precision of floating point numbers in classical computations.

A quantum routine operates a quantum register $|\psi\rangle$ of n qubits placed in an equal superposition of all of its 2^n states. Each of the states is mapped to a parcel $m : |i\rangle \rightarrow (x, y)$. When

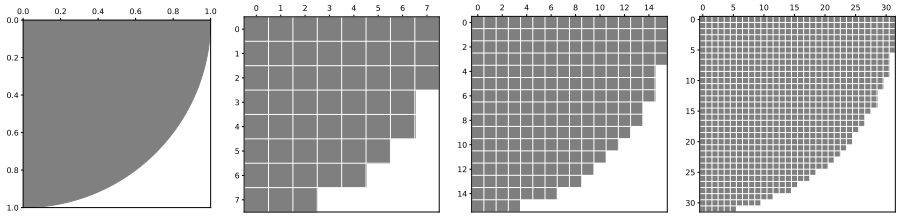

Figure 1: Quarter circle used for the estimation of π with $r = 1$. Left-most figure shows the area within $x^2 + y^2 \leq 1$; the figures to its right show the discretizations with $n = 8, 16$ and 32.

measuring the register, a given state is output, and the map determines whether it accounts for the circle or not. At this point, the Monte Carlo process is the same for the classical and quantum approaches: a random sampler generator and a classifier.

To take advantage of the optimizations provided by QAMC, a new operator W is appended to the previous quantum routine. This operator acts on an extra qubit, which is initialized to state $|0\rangle$, and is set to state $|1\rangle$ when the state of $|\psi\rangle$ corresponds to a parcel that accounts for the circle with multicontrolled-not operators. This new quantum routine can be passed onto the QAMC algorithm, and it will output the estimation of π, with quadratically less shots.

3 Results and conclusions

The proposed method has been tested with several experiments, whose conditions and results are illustrated in Table 1. Each experiment has been performed 100 times, since the probabilistic nature impels to repeat them and consider their averages.

As expected, the classical and the quantum methods present comparable scaling regarding the size of the problem, since both approaches are algorithmically equivalent. This is not the case for the QAMC approach, which yields less shots required for a problem of the same size.

This result showcases the potential for QAMC to optimize real world problems, as long as there is a way to implement them through quantum circuits. This approach will perform the same as quantum hardware improves, and therefore sets a step towards the direction of optimization with Quantum Computing.

Resolution	Precision	Approach		
		Classical	Quantum	QAMC
4x4	10^{-1}	19	24	**7**
	10^{-2}	488	367	**289**
16x16	10^{-1}	22	23	**3**
	10^{-2}	437	476	**276**

Table 1: Average number of shots required by each approach to estimate π up to a given precision $|\pi - \tilde{\pi}| \in \{10^{-1}, 10^{-2}\}$ for 4×4 and 16×16 grid sizes.

References

[1] Ashley Montanaro. Quantum speedup of monte carlo methods. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 471(2181), September 2015.